GARCH TYPE MODELS TO FIND LINKAGES BETWEEN STOCK RETURNS AND INTEREST RATE IN THE CZECH REPUBLIC

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Modely typu Garch na vyhľadanie prepojení medzi návratnosťou akcií a úrokovou sadzbou v Českej republike

Abstract: *The accurate estimation and forecasting of volatility in financial market is a crucial issue and has been a popular subject of research. The aim of the paper is to analyze relationship between the Prague Stock Exchange Price Index of the Czech Republic (PX Index) and interest rate. For this purpose, we employ ordinary as well as asymmetric generalized autoregressive conditionally heteroskedastic (GARCH) methodology with explanatory variable. We assume positive relationship between these variables in terms of volatility. In other words, if the interest rate increases, volatility of PX Index returns should increase as well and vice versa. Our paper provides demonstration that there is a linkage between interest rate and PX Index and thus, that interest rate has explanatory power for PX Index and can help to improve the forecasts of this index.*

Keywords: *PX Index, interest rate, volatility, GARCH, asymmetry*

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1 Introduction

Modeling and forecasting stock market volatility has been the subject of vast empirical and theoretical investigation over the past two decades by academics and practitioners alike. Volatility, as measured by the standard deviation or variance of returns, is often used as a crude measure of the total risk of financial assets. Many value-at-risk models for measuring market risk require the estimation or forecast of a volatility parameter. The volatility of stock market prices also enters directly into the Black-Scholes formula for deriving the prices of traded options.

Region and country economic factors, such as tax and interest rate policy, contribute to the directional change of the market and thus volatility. For example, in many countries, the central bank sets the short-term interest rates for overnight borrowing by banks. When they change the overnight rate, it can cause stock markets to react, sometimes violently. The central banks of countries generally tend to reduce interest rates when they wish to increase investment and consumption in the country's economy.

In our paper we address the question of whether interest rate contain any incremental information useful to explain future volatility of stock returns, particularly PX Index. We have chosen interest rate as explanatory variable of PX Index as it is a crucial economic variable strongly affecting the economic development of each country. Furthermore, there is international empirical evidence demonstrating linkages between stock prices and interest rates.

The reminder of this paper is organized as follows: in the next Section 2, we review the literature. Section 3 introduces the data and the methodology employed. Section 4 presents the empirical results and finally, Section 5 includes summarizes and concludes.

2 Literature review

The relationship between stock prices and interest rates has received considerable attention in the literature. There is empirical evidence in literature that provides support for stock return predictability using macro variables. In the academic literature on stock market predictability, the prevalent view until the 1970s was that stock prices are very closely described by a random walk and that no economically exploitable predictable patterns exist. More recent empirical work, however, reports evidence that stock returns are to some extent predictable. There is empirical evidence showing that stock prices tend to fluctuate with economic news and thus, that macroeconomic variables have explanatory power for stock returns (Rapach and Zhou, 2012). One of the first was Schwert (1989) who underlined, that macroeconomic data can help explain why stock return volatility changes over time. Nowadays, numerous macroeconomic and financial variables are available, but typically only a small number of variables is considered as possible predictors in a return regression. Some studies find strong evidence of ability to predict stock return using particular variable, while others demonstrate contrary results for the same variable. Fama (1990), and Ferson and Harvey (1993), among others, have found short-run relationship among stock returns, macroeconomic and financial variables.

Rapach et al. (2005) use a large set of macroeconomic variables and analyze their impact on stock returns separately and find out that among inflation rate, money stocks, term spread, industrial production and unemployment rate, interest rates are the most consistent and reliable predictors of stock returns across twelve industrialized countries.

One set of studies in the literature, including Fama and Schwert (1977), Campbell (1987), Breen, Glosten, and Jagannathan (1989), and Ferson (1989), examines the relation between short-term stock returns and short-term interest rates. These studies typically find that short interest rates have power to forecast short-term stock returns and/or short-term risk premiums. Zhou (1996) in his paper shows that interest rates have an important impact on stock returns, especially at long horizons. The paper finds that long-term interest rates explain a major part of variation in dividend-price ratios and suggests that the high volatility of the stock market is related to the high volatility of long-term bond yields and may be accounted for by changing forecasts of discount rates. Lee (1992) finds that the volatility of stock returns increases with the level of the risk-free rate, i.e. when rates are high in one period, stock returns are more likely to be volatile in the next.

3 Methodology and Data

3.1 Methodology

While the traditional ARMA-type models assume homoscedasticity, i.e. a constant variance and covariance function, the autoregressive conditional heteroskedastic ARCH model of Engle (1982) was the first formal model which successfully addressed the problem of heteroskedasticity. ARCH models are employed commonly in modeling financial time series that exhibit time-varying volatility clustering, i.e. periods of swings followed by periods of relative calm. A useful generalization of this model is the GARCH parameterization introduced independently by Bollerslev (1986) and Taylor (1986). Since then, many extensions of GARCH-type models have been developed. See Bollerslev et al. (1992) and Bollerslev et al. (1994) for surveys.

In the next section we define ordinary and asymmetric GARCH models with explanatory variable.

GARCH

In GARCH model, the conditional variance σ_t^2 depends on lagged squared errors ε_{t-1}^2, ε_{t-2}^2, … and upon its previous own lags σ_{t-1}^2, σ_{t-2}^2, … . Thus, conditional variance equations in its simplest form GARCH(1,1) can be defined as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{3.1}$$

Since σ_t^2 is a conditional variance, its value must always be strictly positive. A negative variance at any point in time would be meaningless. In order to ensure positive conditional variance estimates, all of the coefficients in the conditional variance are usually required to be non-negative. Thus, non-negativity constraints for our model are in form $\alpha_0>0$ and α_1, $\beta_1 \geq 0$. The model is covariance stationary if and only if $\alpha_1 + \beta_1 < 1$.

Coefficient α_1 expresses the influence of random deviations in the previous period on σ_t and β_1 measures the part of the realized variance in the previous period that is carried over into the current period. The sizes of the parameters α_1 and β_1 determine the short-run dynamics of the resulting volatility time series. Large value of GARCH error coefficient means that volatility reacts intensely to market movements and large value of GARCH lag coefficient indicates that shocks to conditional variance take a long time die out, so volatility is persistent.

The GARCH(1,1) model can be extended to a GARCH(p,q) formulation, where the current conditional variance is parameterized to depend upon q lags of the squared error and p lags for the conditional variance. However, in general a GARCH(1,1) is sufficient and parsimonious to capture the volatility clustering in the data.[2]

Now, to obtain GARCH-X model we need to add explanatory variable to the variance equation. We can rewrite the conditional variance equation as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \delta X_t \tag{3.2}$$

where X_t is explanatory variable, in our case interest rate. Variable X_t has no explanatory power for stock returns if $\delta = 0$. Also, we consider positive sign of δ, as we assume that the increase of interest rate should imply increase of stock prices volatility and vice versa.

GARCH-M Model

As the level of uncertainty in asset returns varies over time, it appears reasonable that risk-averse economic agents require some compensation for holding these assets in times of higher volatility. This compensation is called the risk premium. As long as homoscedasticity assuming models were dominating it was not easy to test such a hypothesis. However, this became much easier with econometric tools that allow the volatility to change over time. Thus, if we include conditional variance or standard deviation in the mean equation, we get the GARCH-in-Mean (GARCH-M) model introduced by Engle et al. (1987). The importance of this family of models is that, unlike the basic ARCH and GARCH models, they portray the fundamental trade-off relationship between expected returns and the volatility measure, with the coefficient γ capturing the dynamic pattern of the changing risk premium over time.

[2] A common way to write the full GARCH model is the following: $y_t = \mu_t + \varepsilon_t$, $\varepsilon_t = \sigma_t e_t$, where σ_t^2 is defined in (3.1). y_t represents a time series value at the time t, and μ is the mean of the GARCH model.

GARCH-M model can be generally written in the following form:

$$y_t = \mu + \gamma log\,(\sigma_t^2) + \varepsilon_t;\ \varepsilon_t \sim N(0,\sigma_t^2) \tag{3.3}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{3.4}$$

We include the conditional variance in the mean equation in a logarithmic form, as it maximizes likelihood function; however it is also possible to use the square root or just untransformed conditional variance σ_t^2. If γ is positive and statistically significant, then increased risk, given by an increase in the conditional variance, leads to rise in the mean return. We would expect a positive coefficient since bearing of higher risk should be rewarded by higher returns.

Asymmetric GARCH models

One of the primary restrictions of GARCH models is that they enforce a symmetric response of volatility to positive and negative shocks. However, it has been argued that a negative shock to financial time series is likely to cause volatility to rise by more than a positive shock of the same magnitude. As noted by Christie (1982), stock price fluctuations are negatively correlated with volatility, which entails more uncertainty and hence generate more volatility. This asymmetric behavior is also known as the leverage effect.

Next, two popular asymmetric models are defined.

The GJR model

The GJR model was introduced by Glosten et al. (1993) and is a simple extension of GARCH with additional term added to account for possible asymmetries. The conditional variance is now given by:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 I_{t-1} \tag{3.5}$$

where $I_{t-1} = 1$ if $u_{t-1} < 0$ and $I_{t-1} = 0$ otherwise.

Condition for non-negativity will be $\alpha_0 > 0$, $\alpha_1 > 0$, $\beta_1 \geq 0$ and $\alpha_1 + \gamma \geq 0$. For leverage effect we would see $\gamma > 0$. Hence, positive news has an impact of α_1 and negative news has an impact of $\alpha_1 + \gamma$, with negative news having a greater effect on volatility if $\gamma > 0$.

Lastly, to obtain GJR model with explanatory variable we get the following equation which is given by:

$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^{\ 2} + \beta_1\sigma_{t-1}^{\ 2} + \gamma\varepsilon_{t-1}^{\ 2}\, I_{t-1} + \delta X_t \tag{3.6}$$

The EGARCH model

The exponential GARCH (EGARCH) model was proposed by Nelson (1991), in order to capture the leverage effect. The specification for the conditional variance is given by following formula,

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 ln\sigma_{t-1}^2 \tag{3.7}$$

The presence of leverage effect can be tested by the hypothesis that $\gamma<0$. The impact is asymmetric if $\gamma \neq 0$. This model is successful because, except that it captures the leverage effect, no inequality constraints need to be imposed on the model parameter. Since the ln (σ_t^2) is modeled, even if parameters are negative, σ_t^2 will always be positive.

Specification for EGARCH model with explanatory variable is as follows,

$$\ln(\sigma_t^2) = \alpha_0 + \alpha_1 \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1 ln\sigma_{t-1}^2 + \delta X_t \tag{3.8}$$

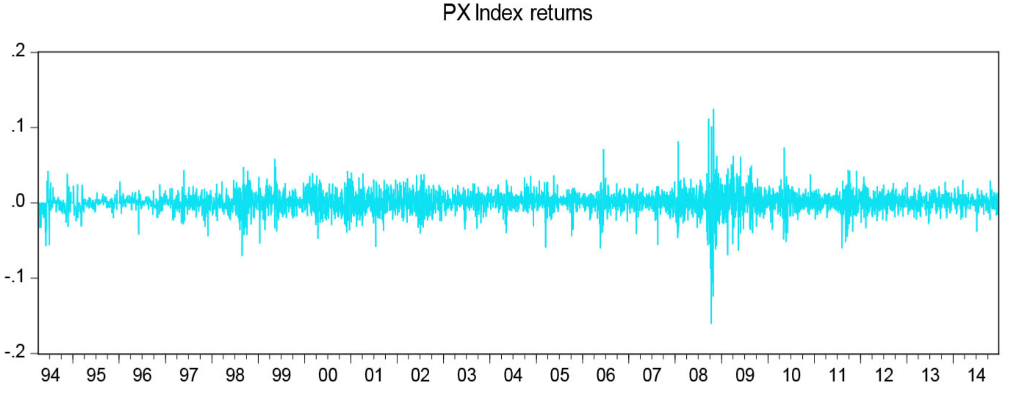
3.2 Data

Our dataset consists of 5405 daily observations for PX Index and Czech Republic Interbank Overnight Interest Rate covering period from 06/04/1994 to 23/12/2014. Data were obtained from Datastream. For Overbank Interest Rate are used changes in our paper. Prices of the index P_t are converted to returns r_t by calculating the price log difference with the following formula,

$$r_t = \ln(P_t/P_{t-1}) \tag{3.9}$$

Figure 1 shows clearly that the mean is constant and around zero, but the variance changes over time showing evidence of volatility clustering.

Figure 1: Daily returns for PX Index and daily interest rate covering period from 06/04/1994 to 23/12/2014.



Source: Author

Table 1 represents the descriptive statistics for the PX Index returns series. The mean and the median are consistent with zero value. The negative skewness of PX Index indicates asymmetric distribution and kurtosis statistics show the leptokurtic characteristic of the return distribution. The existence of non-normality is supported also by Jarque-Bera test statistic which points out that the null hypothesis of normal distribution should be rejected. Instead of normal distribution, Student's t-distribution with degree of freedom fixed at 10 is used for all models.

Table 1: Descriptive statistics for PX Index.

Mean	Median	Max	Min	Std.Dev.	Skewness	Kurtosis	J.-Bera
-9.18e-06	0.00000	0.123641	-0.161855	0.013541	-0.443658	15.15237	33429.93

Source: Author's calculations

To consider whether an ARCH effect appears in our series we have to test for the presence of conditional heteroskedasticity. For this purpose the Ljung-Box test and the Lagrange Multiplier test are carried out. As Ljung-Box Q(m) statistics of PX Index returns are significant with p-value equal to zero, it indicates that squared residuals are autocorrelated. Also, according to the Lagrange Multiplier the null hypothesis of homoskedasticity is clearly rejected at 1% significant level with p-value equal to zero, indicating the presence of ARCH effect in the return series.

4 Empirical results

As shown in the previous section, daily returns exhibit volatility clustering and fat tails. For this reason, we use family of GARCH models as they have been proven to capture these characteristics.

For the mean equation ARMA(1,2) structure with constant is used in all models. Coefficients are statistically significant at 1% level. The in-mean term represented by $\log(\sigma_t^2)$ is significant in almost all models; however, the sign is negative, instead of our hypothesis to be positive, i.e. higher variance of the PX index is associated with lower returns. This result is contradictory according to the idea of risk premium for higher returns. We therefore do not use in-mean term for further analysis and these results are not reported.

Table 2 reports the parameter estimates of the alternative GARCH family models defined in previous part. The estimates of GARCH model show that all coefficients of the mean and variance equation are statistically significant at 1% level and satisfy the non-negativity constraints. The sum of ARCH and GARCH term, i.e. $\alpha_1 + \beta_1$ is less than one, but very close to unity, which indicates that shocks to volatility are highly persistent. Residual diagnostics, however, shows that residuals are autocorrelated. Accounting with interest rate as exogenous variable in GARCH model we found this variable insignificant.

Finally, in order to test whether there are any remaining ARCH effects in the standardized residuals, a Lagrange Multiplier (LM) test is carried out. Results show that the null hypothesis of no ARCH effect cannot be rejected, indicating no remaining ARCH effect in GARCH and GARCH-X models.

In GJR and GJR-X models, $\alpha_1 + \gamma > 0$ holds indicating that bad news increase the conditional volatility and have much greater impact on conditional variance than good news α_1, showing a substantial asymmetric effect. When the information of interest rate, significant at 6% level, is added the log-likelihood as well as R-squared is higher than in GJR model. Shocks to volatility remains highly persistent. The residuals are not autocorrelated and *p*-value of LM tests for ARCH effect is not significant indicating that variance equations are correctly specified in both GJR and GJR-X models.

The impact of interest rate on PX index in the conditional volatility can also be found by comparing EGARCH and EGARCH-X model. The coefficient of logarithmic changes in interest rate is found to be statistically significant at 2% level. Similar to the results from the GJR models, the log-likelihood test shows that the information contained in interest rate has incremental explanatory power for conditional volatility. The standardized residuals are not autocorrelated and exhibit no remaining ARCH effect.

R-squared statistics has a small value for all fitted models; however, as theory tells us the predictive component in stock returns is normally small and *R*-squared statistics below 1% can be economically relevant.

In sum, asymmetric models are superior to ordinary GARCH model. Furthermore, including the interest rate as explanatory variable in the model contains incremental information useful for explaining the conditional volatility and thus, we can summarize that interest rate has positive effect on PX index returns volatility.

Table 2: Estimation of GARCH family models.

	GARCH	GARCH-X	GJR	GJR-X	EGARCH	EGARCH-X
α_0	3,19E-06 (0,0000)	2,62E-06 (0,0000)	2,63E-06 (0,0000)	2,99E-06 (0,0000)	-0,404836 (0,0000)	-0,408458 (0,0000)
α_1	0,128365 (0,0000)	0,127973 (0,0000)	0,082697 (0,0000)	0,086471 (0,0000)	0,230177 (0,0000)	0,231849 (0,0000)
β_1	0,854679 (0,0000)	0,857477 (0,0000)	0,861154 (0,0000)	0,855105 (0,0000)	0,974855 (0,0000)	0,974576 (0,0000)
γ	- -	- -	0,081738 (0,0000)	0,080126 (0,0000)	0,052860 (0,0000)	0,053837 (0,0000)
δ	- -	1,12E-05 (0,2084)	- -	1,80E-05 (0,0509)	- -	0,304556 (0,0144)
Log-L	16763,42	16769,54	16784,63	16785,87	16798,44	16800,69
R^2	0,011408	0,011264	0,011171	0,012463	0,011811	0,011931
Note: *p-values are reported in brackets.*						

Source: Author

5 Conclusion

Modeling and forecasting volatility is an important issue in financial market and there is an extensive research agenda that reflects the importance of volatility of investment, option pricing and risk management. Thus, an accurate estimation of the volatility of asset prices is crucial for assessing investment risk.

This paper provides a comparative evaluation of GARCH family models and analyzes relationship between stock price index and interest rate as was proposed in many previous studies. In particular, we analyze if there is relationship between PX Index and Czech Republic Interbank Overnight Interest Rate. In the paper we applied ordinary and asymmetric GARCH family models to account for typical characteristics of financial stock market returns, such as volatility clustering effect, the leverage effect and leptokurtic distribution. The use of in-mean term results to be not appropriate as we found this term to be negative in all models. In all models we found effect of the interest rate on stock returns volatility to be positive, meaning that higher interest rate is associated with higher stock market volatility.

To summarize, the results suggest that interest rate does contain additional information useful for the future Czech stock market volatility. The presence of the asymmetric effect is very important as it significantly improves the performance of GARCH family models. Overall, a model that includes both asymmetric effect and interest rate as explanatory variable is superior over other models fitted in this paper and can contribute to improve estimates of stock market volatility.

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