EQUILIBRIUM OF THE DUOPOLY WITH LINEAR UTILITY FUNCTION OF THE DEMAND

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Rovnováha duopolu s lineárnou úžitkovou funkciou dopytu

Abstract: *We have identified the limitation of the model of the market with linear utility function of the demand. Model of the market with perfectly substitutive products does not correspond to real world longterm markets, because it leads to negative profit for at least one of the companies. This negative profit is defined by the equilibrium of the market that we have derived from the best response functions of the companies by two concepts, the Nash equilibrium in pure strategies approach and the modeling of the considerations of the companies approach. Both concepts lead to the same outputs. We obtained the best response functions from the maximization of the profit of the particular companies and from the particular demand functions for the products. We were able to prove that the rational company with relatively higher or equal variable cost will always set the price of the product to the level of the variable costs. We suggest other utility function, e.g. quasilinear, to be used to analyze the competition on real long lasting markets.*

Keywords: *Competition, duopoly, Linear utility function, perfect substitutes, demand function, best response function, optimal Pricing, equilibrium, Nash equilibrium.*

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1 Introduction

Duopoly is one of the simplest forms of the competition on the market. It is the situation where two competitive companies are producing two products that meet some specific need of the demand and only these two products are able to meet this specific need of the demand. In other words, we assume these products to be substitutes and the only substitutes for the specific need, and we call the market of these products the duopoly.

Demand has limited resources to spend on the market of the duopoly, so these two companies are competing with their products for this limited demand. This competition is usually driven by the price of the products, because the demand makes its decision based on the price of the particular product. Naturally, we expect that the consumption of the particular product decreases when its price rises and vice versa, because the demand makes the decision of the consumption based on the price of the substitutes. In economy, this expectation is also called the law of demand (Amir, Erickson and Jin, 2015).

The limited resources of the demand are often called the budget of the demand. The set of all combinations of various amounts of products that are subject of selection of the consumer (or whole demand), where the limited resources that demand would like to spend equals the sum of the amounts of products times their corresponding prices, is called the budget constraint (Varian, 1992; 1995).

Economists usually express the motivation of the demand to buy the particular product by utility functions. Utility function is the function that assigns numerical value to every consumer basket made from various number of the products and this function maintain the order of the preferences between various baskets of the demand. Consumer basket is the complete list of the products that are subject to the selection of the consumer (or whole demand) and their quantities. Preferences are simply the expressions of the consumer's opinion (or those of the entire demand) which consumer basket is preferred when comparing any two consumer baskets. (Varian, 1992; 1995)

This article discusses the model of the duopoly that could be applied in the analyses of the real world markets. Therefore, all numerical values in this article are from the set of real numbers.

2 Linear Utility Function (LUF)

Consider two competitive companies $I = \{1, 2\}$ producing two substitutive products and assume that these two products (let us denote the product of the company 1 as product 1 and the product of the company 2 as product 2, so the set of the products will be $I = \{1, 2\}$ as well) are the only products that meet the particular need of the demand. Let us call this particular need of the demand, consumption and production of the products 1 and 2 the market M.

Let $x_i \geq 0$ be the number of units of the product$i \in I$, let $f_i > 0$[2] be the fixed cost of the company $i \in I$, let $c_i > 0$[3] be the variable cost of the production of the unit of the product $i \in I$ and let $p_i \geq 0$ be the price of the unit of the product $i \in I$. Let us assume that the companies are interested only in maximization of their profit and let us define the profit $\pi_i = p_i x_i - f_i - c_i x_i$ for every company $i \in I$. We can see that for any given x_i, f_i and c_i profit of the company i, $\pi_i \geq -f_i <=> p_i \geq c_i$. If $p_i < c_i$ then for any given $x_i > 0$, f_i and $c_i > 0$ profit of the company i $\pi_i < - f_i$, so it is rational to assume $p_i \geq c_i$, $\forall i \in I$. Only in the case when $p_i = c_i$ let us soften the assumption that the company $i \in I, \forall i \in I$ is interested only in maximization of its profit π_i and in this case let us assume that the company $i \in I$, $\forall i \in I$ would produce demanded amount of the product $i \in I$ despite the fact that the company $i \in I$ would be indifferent in this case[4].

Consider total available financial resources of the demand $I_T > 0$. Consider $x_0 \geq 0$ to be number of units of all other products out of the market M that meet any other need of the demand, the units x_0 are equivalent to the units x_i, $i \in I$. Consider $p_0 \geq 0$ to be the price of the unit of the x_0, so we can say that p_0 is the weighted average price of the products outside of the market M in terms of equivalent units of the market M. Let us assume that p_0 and x_0 are not reacting to the situation on the market M, so from the market M point of view the p_0 and x_0 are constants. That means the product $p_0 * x_0$ defines the financial amount that demand spends outside of the market M. That means the differen-

[2] This is a relatively natural requirement, because standardly there are some fixed costs for the companies that are willing to produce.
[3] This is a rational assumption, because when the variable costs are 0 then the company can set the price of the product to 0 what can lead to unlimited consumption of the demand and the market will disappear.
[4] In the case when $p_i = c_i$, $i \in I$ we can assume that although company $i \in I$ is indifferent in producing or not producing in the terms of its profit π_i, company $i \in I$ can prefer to produce. This decision can maintain its human workforce in practice; otherwise, the workforce can lose the practice. It could be more costly to return it back to production (this weak assumption is supported by the assumption that the c_i contains all variable costs including costs of the maintenance and repairs from the wear and tear caused by the production of the product i).

ce $I_T - p_0 x_0 = I_M$ are financial resources that demand spends on the market M and let us require $I_M > 0$, which means $p_0 x_0 < I_T$.

Since we assume products 1 and 2 to be substitutes, let us consider them as the perfect substitutes, so let us assume that utility function of the market M will be linear[5]:

$$U_{L(x_1,x_2)} = A_1 x_1 + A_2 x_2 + D x_0 \text{ with budget constraint } I_M = p_1 x_1 + p_2 x_2, \quad (2.1)$$

under the conditions

$$A_i > 0, D > 0, x_i \geq 0, c_i > 0, p_i \geq c_i, x_0 \geq 0, p_0 \geq 0, I_T > 0, I_M = I_T - p_0 x_0 > 0 \; \forall i \in I,$$

where $A_i > 0$[6] is the attractiveness of the product $i \in I$ for the demand and $D > 0$[7] is the attractiveness of the products outside of the market M and $I_M = p_1 x_1 + p_2 x_2$ is the budget constraint of the demand for the market M. The attractiveness coefficient A_i expresses any kind of attractiveness like quality, marketing or knowingness of the product $i \in I$ in one coefficient.

The assumption that products 1 and 2 are perfect substitutes implies that even if A_1 and A_2 were not equal, the certain amount of the product 1 is considered by the demand to have the same utility as the certain amount of the product 2 and vice versa.

[5] In economic literature, for example in Nechyba (2010) the utility function of the perfect substitutes in duopoly is often expressed as linear function $U_L = x_1 + x_2$. Adding attractiveness coefficients A_i allows us to look at the situation in a more general way. It allows us to analyze changes on market M under changes of the attractiveness of the products 1 and 2. Adding the component Dx_0 does not make the difference in calculation of the market outcomes while we consider the effective duopoly on the market. But the component Dx_0 would allow us to analyze (under some assumptions) the situation, when the effective duopoly would vanish, when one of the company would gain monopoly position (so the component Dx_0 would allow us to deal with the specific extreme situation on market).

[6] We will require the attractiveness of the products to be greater than 0, because in the case when one of the attractiveness of the products equals 0, this product does not belong to the market M anymore. In one special case we can allow the attractiveness of some product to be equal 0. This will be the case when some product loses its attractiveness on the market (maybe by differentiation of the product of the competitor, which could mean huge investments into the quality of the product of the competitor), making the market M the monopoly of the product of the competitor.

[7] We will require the attractiveness of the products outside of the market M to be greater than 0, because otherwise the products on the market M will be the only products that meet all needs of the demand.

3 Demand Functions from LUF

Let us introduce the assumption that the companies are able to produce any demanded amount of their corresponding products. Therefore, there will not be any production constraints in our analyses. So the companies $i \epsilon I$ are free in setting the price $p_i \geq c_i$, $\forall i \epsilon I$ of the products $i \epsilon I$.

Now let us derive the demand functions of the products 1, 2 as maximization of the utility function of the demand under the condition of budget constraint. To do this, we will use the Lagrange method for finding the local maxima and minima of a function subject to equality constraints (Dixit, 1990). The Lagrange function for the linear utility function (2.1) will be

$$\mathcal{L}_L(x_1, x_2, \lambda) = A_1x_1 + A_2x_2 + Dx_0 + \lambda[I_m - p_1x_1 - p_2x_2], \qquad (3.1.1)$$

where λ is the Lagrange multiplier.

The Lagrange method will define three conditions for the local extreme that have to be satisfied in the same time:

$$\frac{d\mathcal{L}_L}{dx_1} = A_1 - \lambda p_1 = 0, \qquad (3.1.2)$$

$$\frac{d\mathcal{L}_L}{dx_2} = A_2 - \lambda p_2 = 0, \qquad (3.1.3)$$

$$\frac{d\mathcal{L}_L}{d\lambda} = I_m - p_1x_1 - p_2x_2 = 0. \qquad (3.1.4)$$

If we consider only the conditions (3.1.2) and (3.1.3), we will get

$$p_1 = \frac{A_1}{A_2}p_2, \text{ or } p_2 = \frac{A_2}{A_1}p_1. \qquad (3.1.5)$$

We can understand the condition (3.1.5) as information that the demand will divide its consumption on market M between products 1 and 2 in the way that $x_1 > 0$ and $x_2 > 0$ <=> (3.1.5). Otherwise one of the following statement has to be in force $x_1 = 0$ or $x_2 = 0$, which implies in the case $x_1 = 0$ that $x_2 = \frac{I_M}{p_2}$ or in the case $x_2 = 0$ that $x_1 = \frac{I_M}{p_1}$, because the demand would spend its financial resources only for the relatively cheaper product in terms of price and attractiveness, what means if $p_1 > \frac{A_1}{A_2}p_2$ then $x_1 = 0$ and if $p_2 > \frac{A_2}{A_1}p_1$ then $x_2 = 0$.

From the foregoing considerations and the Lagrange method for finding the local extreme, we get the demand functions:

$$x_1(p_1,p_2) = \begin{cases} 0 & if\ p_1 > \frac{A_1}{A_2}p_2 \\ \frac{I_M}{2p_1} = \frac{A_2 I_M}{2A_1 p_2} & if\ p_1 = \frac{A_1}{A_2}p_2, \\ \frac{I_M}{p_1} & if\ p_1 < \frac{A_1}{A_2}p_2 \end{cases} \quad (3.2.1)$$

$$x_2(p_1,p_2) = \begin{cases} 0 & if\ p_2 > \frac{A_2}{A_1}p_1 \\ \frac{I_M}{2p_2} = \frac{A_1 I_M}{2A_2 p_1} & if\ p_2 = \frac{A_2}{A_1}p_1 \\ \frac{I_M}{p_2} & if\ p_2 < \frac{A_2}{A_1}p_1 \end{cases} \quad (3.2.2)$$

4 Best Response Functions from LUF

Let us assume that the companies are interested only in maximization of their profit and that they are rational.

Let us add another assumption that both companies have full information of the values A_i, D, I_M, f_i, $c_i \forall_i \epsilon I$ so they both know (3.2.1) and (3.2.2), and they can observe the price $p_i \forall_i \epsilon I$.

Let us look at the maximization problem of the profit of both companies, that is $\forall_i \epsilon I$:

$$\max \pi_i(p_i) = p_i x_i - f_i - c_i x_i. \quad (4.1)$$

When we *fix* $p_2 = p_2^{fix}$, company 1 would never set $p_1 > \frac{A_1}{A_2}p_2^{fix}$, until $c_1 \leq \frac{A_1}{A_2}p_2^{fix}$, when company 1 would set $p_1 = c_1$.[8] Setting $p_1 > \frac{A_1}{A_2}p_2^{fix}$ by the company 1 would mean zero demand for the product 1, so $x_1 = 0$, what would mean the minimal profit for the company 1. If $c_1 \leq \frac{A_1}{A_2}p_2^{fix}$ company 1 would set $p_1 = c_1$ although there will be zero demand for the product 1, because this is the minimum price when the company 1 is willing to produce[9]. We can equally consider behavior of the company 2 when trying to set

[8]The requirement that company 1 would set the price $p_1 = c_1$ in the case when $c_1 \leq \frac{A_1}{A_2}p_2^{fix}$ comes from the condition that $p_1 \geq c_1$ and the assumption that in the case $p_1 = c_1$ company 1 would prefer to produce demanded amount of the product 1, although company 1 is indifferent in producing or not producing in the terms of its profit π_1.
[9] Assumption that the company 1 prefers to produce any demanded amount $x_1 \geq 0$ in the case when $p_1 = c_1$, although company 1 is indifferent in producing or not producing in the terms of profit π_1.

$p_2 > \frac{A_2}{A_1} p_1^{fix}$ if we fix $p_1 = p_1^{fix}$. Company 2 will never set $p_2 > \frac{A_2}{A_1} p_1^{fix}$ (only in the case when $c_2 > \frac{A_2}{A_1} p_1^{fix}$ and in this case $p_2 = c_2$ and $x_2 = 0$), because it would lead to zero demand for the product 2 ($x_2 = 0$), which would mean the minimal profit for the company 2 ($\pi_2 = -f_2$).

Now, let us consider that company 1 (when p_2 is fixed) sets $p_1 > \frac{A_1}{A_2} p_2^{fix}$ under the condition $c_1 \leq \frac{A_1}{A_2} p_2^{fix}$ [10] In this case, the demand is divided between the products 1 and 2 (what means $x_1 > 0$ and $x_2 > 0$) and $\pi_1 = \frac{p_1 - c_1}{2p_1} I_M - f_1$ For $p_1 > c_1$ company 1 will have $\pi_1 > -f_1$, so with the decision $p_1 > \frac{A_1}{A_2} p_2^{fix}$ company 1 can reach higher than minimal profit. Analogically, if company 2 (when p_1 is fixed) sets $p_2 > \frac{A_2}{A_1} p_1^{fix}$ under the condition $c_2 > \frac{A_2}{A_1} p_1^{fix}$, then $\pi_2 = \frac{p_2 - c_2}{2p_2} I_M - f_2$. For $p_2 > c_2$ company 2 will have $\pi_2 > -f_2$, so with the decision $p_2 > \frac{A_2}{A_1} p_1^{fix}$ company 2 can reach higher than minimal profit.

Let us fix p_2 again and assume that company 1 would like to set $p_1 > \frac{A_1}{A_2} p_2^{fix}$. Remember that p_1 is rational only if $p_1 \geq c_1$. If both assumptions are in force, then $x_1 = \frac{I_M}{p_1}$, so $\pi_1 = \frac{p_1 - c_1}{p_1} I_M - f_1$ and if $p_1 > c_1$ then $\pi_1 > -f_1$. So the decision $p_1 > \frac{A_1}{A_2} p_2^{fix}$ can lead to higher than minimal profit.

Analogically, if company 2 (when p_1 is fixed) sets $p_2 > \frac{A_2}{A_1} p_1^{fix}$ under condition $p_2 \geq c_2$, then $\pi_2 = \frac{p_2 - c_2}{p_2} I_M - f_2$ and if $p_2 > c_2$ then $\pi_2 > -f_2$. So the decision $p_2 > \frac{A_2}{A_1} p_1^{fix}$ of the company 2 can lead to higher than minimal profit.

But what is rational for the company 1? To set the $p_1 = \frac{A_1}{A_2} p_2^{fix}$ or to set $p_1 < \frac{A_1}{A_2} p_2^{fix}$? Let us transform this question into the question, when is company 1 willing to set $p_1 < \frac{A_1}{A_2} p_2^{fix}$. Let us denote $p_1^* = \frac{A_1}{A_2} p_2^{fix}$ and $\pi_1^*(p_1^*) = \frac{p_1^* - c_1}{2p_1^*} I_M - f_1$ and let us introduce parameter $\Delta > 0$. Then let us define for the price $p_1^L < \frac{A_1}{A_2} p_2^{fix}$ equation $p_1^L = p_1^* - \Delta$, so the value of the $\Delta > 0$ can define any $p_1^L < \frac{A_1}{A_2} p_2^{fix}$. Then the profit π_1^L of the company 1 under the conditions $p_1^L < \frac{A_1}{A_2} p_2^{fix}$, $p_1^L = p_1^* - \Delta$, can be rewritten to $\pi_1^L = \frac{p_1^* - \Delta - c_1}{p_1^* - \Delta} I_M - f_1$. So we get the question when $\pi_1^L > \pi_1^*$, or

$$\frac{p_1^* - \Delta - c_1}{p_1^* - \Delta} I_M - f_1 > \frac{p_1^* - c_1}{2p_1^*} I_M - f_1 \qquad (4.2.1)$$

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[10] In the case that the $c_1 > \frac{A_1}{A_2} p_2$ company can not set the price $p_1 = \frac{A_1}{A_2} p_2$ because then the $p_1 < c_1$, which is against the assumption $p_1 \geq c_1$, so the company 1 can set the price $p_1 = \frac{A_1}{A_2} p_2$ only if $c_1 \leq \frac{A_1}{A_2} p_2$.

and from there

$$\Delta < \frac{\frac{A_1}{A_2}p_2^{fix}\left(\frac{A_1}{A_2}p_2^{fix}-c_1\right)}{\frac{A_1}{A_2}p_2^{fix}+c_1}. \qquad (4.2.2)$$

Substituting $\Delta = p_1^{*} - p_1^{L}$ and $p_1^* = \frac{A_1}{A_2}p_2^{fix}$ we will get from (4.2.2) condition for $p_1^L < \frac{A_1}{A_2}p_2$ to be preferred by company 1 to $p_1^* = \frac{A_1}{A_2}p_2$:

$$p_1^L > \frac{2\frac{A_1}{A_2}c_1 p_2^{fix}}{\frac{A_1}{A_2}p_2^{fix}+c_1}. \qquad (4.2.3)$$

Combining (4.2.3) and $p_1^L < \frac{A_1}{A_2}p_2^{fix}$ we will get the condition for existence of the price $p_1^L < \frac{A_1}{A_2}p_2^{fix}$ that is preferred by company 1 $p_1^* = \frac{A_1}{A_2}p_2^{fix}$, so from

$$\frac{2\frac{A_1}{A_2}c_1 p_2^{fix}}{\frac{A_1}{A_2}p_2^{fix}+c_1} < p_1^L < \frac{A_1}{A_2}p_2^{fix},$$

we will get

$$\frac{2\frac{A_1}{A_2}c_1 p_2^{fix}}{\frac{A_1}{A_2}p_2^{fix}+c_1} < \frac{A_1}{A_2}p_2^{fix},$$

so we get

$$c_1 < \frac{A_1}{A_2}p_2^{fix}. \qquad (4.2.5)$$

So if $c_1 < \frac{A_1}{A_2}p_2^{fix}$ then there exists $p_1^L < \frac{A_1}{A_2}p_2^{fix}$ that is strictly preferred by company 1 to $p_1^* = \frac{A_1}{A_2}p_2^{fix}$ when the p_2^{fix} is given.

So if $c_1 < \frac{A_1}{A_2}p_2^{fix}$ any price that meets $p_1 > \frac{2\frac{A_1}{A_2}c_1 p_2^{fix}}{\frac{A_1}{A_2}p_2^{fix}+c_1} \wedge p_1 < \frac{A_1}{A_2}p_2^{fix}$ is preferred by the company 1 to $p_1^* = \frac{A_1}{A_2}p_2^{fix}$ when the p_2^{fix} is given.

Analogical consideration about the decision of the company 2 in setting the price $p_2^L < \frac{A_2}{A_1}p_1^{fix}$ or $p_2^* = \frac{A_2}{A_1}p_1^{fix}$ when the p_1^{fix} is given would lead us to: if $c_2 < \frac{A_2}{A_1}p_1^{fix}$ any price that meets $p_2 > \frac{2\frac{A_2}{A_1}c_2 p_1^{fix}}{\frac{A_2}{A_1}p_1^{fix}+c_2} \wedge p_2 < \frac{A_2}{A_1}p_1^{fix}$ is preferred by company 2 to $p_2^* = \frac{A_2}{A_1}p_1^{fix}$ when the p_1^{fix} is given.

If $c_1 = \frac{A_1}{A_2}p_2^{fix}$ then the company 1 would set $p_1 = c_1 = \frac{A_1}{A_2}p_2^{fix}$ and analogically if $c_2 < \frac{A_2}{A_1} p_1^{fix}$ then company 2 would set $p_2 = c_2 = \frac{A_2}{A_1} p_1^{fix}$.

The best response function of the company 1 to the p_2 will be

$$p_1(p_2) = \begin{cases} c_1 & if\ c_1 > \frac{A_1}{A_2} p_2 \\ c_1 = \frac{A_1}{A_2} p_2 & if\ c_1 = \frac{A_1}{A_2} p_2 \\ \frac{A_1}{A_2} p_2 - \Delta_1, \Delta_1 > 0 \wedge \Delta_1 < \frac{\frac{A_1}{A_2} p_2 \left(\frac{A_1}{A_2} p_2 - c_1\right)}{\frac{A_1}{A_2} p_2 + c_1}, \Delta_1 = \min_{\Delta > 0} \Delta & if\ c_1 < \frac{A_1}{A_2} p_2 \end{cases} \quad (4.3.1)$$

and the best response function of the company 2 to the p_1 will be

$$p_2(p_1) = \begin{cases} c_2 & if\ c_2 > \frac{A_2}{A_1} p_1 \\ c_2 = \frac{A_2}{A_1} p_1 & if\ c_2 = \frac{A_2}{A_1} p_1 \\ \frac{A_2}{A_1} p_1 - \Delta_2, \Delta_2 > 0 \wedge \Delta_2 < \frac{\frac{A_2}{A_1} p_1 \left(\frac{A_2}{A_1} p_1 - c_2\right)}{\frac{A_2}{A_1} p_1 + c_2}, \Delta_2 = \min_{\Delta > 0} \Delta & if\ c_2 < \frac{A_2}{A_1} p_1 \end{cases} . \quad (4.3.2)$$

5 Equilibrium of the Market M with LUF

Once we have defined the best response function for the companies on the market M, we are able to identify the equilibrium, so the unit prices of the particular products that the rational companies that maximize their profits would set facing the situation defined by the market M.

To do this, we can use two concepts. One concept is based on the Nash equilibrium in pure strategies, the concept used in the Game theory for finding the optimal strategies for players in particular games (Fudenberg and Tirole, 1991). Nash equilibrium in pure strategies defines the optimal strategy (the unit price of the particular product in the case of the market M) for every player (company) when taking into the consideration the rationality of all players (companies) in the game (on the market M).[11] Second concept is based on the modeling of the particular rational considerations of the companies. Both concepts lead to the same output, the same equilibrium unit prices of the particular products where neither company can reach higher profit by one-side deviation.

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[11] We use here the Game theory terminology.

5.1 Nash equilibrium approach

The Nash equilibrium in pure strategies is the profile of the strategies of all players in the game, where the strategy of every player is the best reaction to the strategies of every other player in the game (Fudenberg and Tirole, 1991). Therefore, in the case of our market M the Nash equilibrium in pure strategies is defined by the combination of the best response functions (4.3.1) and (4.3.2).

The company $i \in I$ can face three possible situations on the market M that are defined by the relative unit variable costs in comparison to the unit variable costs of its competitor. Let $-i \in I, -i \neq i$ be the competitor of the company i on the market M. We say that the company i has relatively higher unit variable costs if $c_i > \frac{A_i}{A_{-i}} c_{-i}$, relatively equal unit variable costs if $c_i = \frac{A_i}{A_{-i}} c_{-i}$ and relatively lower unit variable costs if $c_i < \frac{A_i}{A_{-i}} c_{-i}$.

Every rational company $i \in I$ can set the unit price p_i only in the way that $p_i \geq c_i$.

Let us assume that the company i has relatively lower unit variable costs, so $c_i < \frac{A_i}{A_{-i}} c_{-i}$. Company $-i$ will never set the unit price $p_{-i} > c_{-i}$, because there will always be the unit price $p_i = \frac{A_i}{A_{-i}} p_{-i} - \Delta_i, \Delta_i > 0 \wedge \Delta_i < \frac{\frac{A_i}{A_{-i}} p_{-i}\left(\frac{A_i}{A_{-i}} p_{-i} - c_i\right)}{\frac{A_i}{A_{-i}} p_{-i} + c_i}$, $\Delta_i = \min_{\Delta > 0} \Delta$, what would be the rational decision of the company i, what would lead to the situation where whole demand buys the product i, which means zero demand for the product $-i$, so the negative profit for the company $-i$. So the company $-i$ would set $p_{-i} = c_{-i}$. The best response of the company i to this is to set the

$$p_i = \frac{A_i}{A_{-i}} c_{-i} - \Delta_i, \Delta_i > 0 \wedge \Delta_i < \frac{\frac{A_i}{A_{-i}} c_{-i}\left(\frac{A_i}{A_{-i}} c_{-i} - c_i\right)}{\frac{A_i}{A_{-i}} c_{-i} + c_i}, \Delta_i = \min_{\Delta > 0} \Delta.$$

So we get the equilibrium, the Nash equilibrium in pure strategies,

$$p_i = \frac{A_i}{A_{-i}} c_{-i} - \Delta_i, \Delta_i > 0 \wedge \Delta_i < \frac{\frac{A_i}{A_{-i}} c_{-i}\left(\frac{A_i}{A_{-i}} c_{-i} - c_i\right)}{\frac{A_i}{A_{-i}} c_{-i} + c_i}, \quad \Delta_i = \min_{\Delta > 0} \Delta \quad \text{and } p_{-i} = c_{-i}.$$

Now, let us assume that the company i has relatively equal unit variable costs, so $c_i = \frac{A_i}{A_{-i}} c_{-i}$. Company $-i$ will never set the unit price $p_{-i} > c_{-i}$, because there will always be the unit price

$$p_i = \frac{A_i}{A_{-i}} p_{-i} - \Delta_i, \Delta_i > 0 \wedge \Delta_i < \frac{\frac{A_i}{A_{-i}} p_{-i}\left(\frac{A_i}{A_{-i}} p_{-i} - c_i\right)}{\frac{A_i}{A_{-i}} p_{-i} + c_i}, \quad \Delta_i = \min_{\Delta > 0} \Delta,$$

which would be the rational decision of the company i, which would lead to the situation where whole demand buys the product i, what means zero demand for the product $–i$, so the negative profit for the company $–i$. So the company $–i$ would set $p_{-i} = c_{-i}$. The best response of the company i to this is to set $p_i = c_i$. So we get the equilibrium, the Nash equilibrium in pure strategies, $p_i = c_i$ and $p_{-i} = c_{-i}$.

Finally, let us assume that the company i has relatively higher unit variable costs, so $c_i > \frac{A_i}{A_{-i}} c_{-i}$. This is actually the opposite case to the situation when the company i has relatively lower unit variable costs. Analogically, it leads to the equilibrium, the Nash equilibrium in pure strategies, $p_i = c_i$ and

$$p_{-i} = \frac{A_{-i}}{A_i} c_i - \Delta_{-i}, \Delta_{-i} > 0 \land \Delta_{-i} < \frac{\frac{A_{-i}}{A_i} c_i \left(\frac{A_{-i}}{A_i} c_i - c_{-i} \right)}{\frac{A_{-i}}{A_i} c_i + c_{-i}}, \Delta_{-i} = \min_{\Delta > 0} \Delta.$$

5.2 Modeling of the considerations of the companies' approach

The concept based on the modeling of the particular rational considerations of the companies is slightly more complex to prove. It is based on the motivation of every company to set slightly lower unit price of its product to the relative unit price of the competitor's product, which is the unit price of the competitor's product times the rate of the attractiveness of the products.

This motivation is implied by the best response functions (4.3.1) or (4.3.2). Any company i facing any unit price of the competitor's product p_{-i} on the market M is motivated to set its price to

$$p_i = \frac{A_i}{A_{-i}} p_{-i} - \Delta_i, \Delta_i > 0 \land \Delta_i < \frac{\frac{A_i}{A_{-i}} p_{-i} \left(\frac{A_i}{A_{-i}} p_{-i} - c_i \right)}{\frac{A_i}{A_{-i}} p_{-i} + c_i}, \Delta_i = \min_{\Delta > 0} \Delta,$$

until $p_i \geq c_i$. Once there is no Δ_i that meets

$$\Delta_i > 0 \land \Delta_i < \frac{\frac{A_i}{A_{-i}} p_{-i} \left(\frac{A_i}{A_{-i}} p_{-i} - c_i \right)}{\frac{A_i}{A_{-i}} p_{-i} + c_i},$$

company i will set $p_i = c_i$.

Companies are setting the unit prices of their products simultaneously, so they cannot observe the unit price of the competitor's product before their decision. But this does not mean that the companies cannot make rational expectations about the unit price of the competitor's product.

We can model the considerations of the companies by the successive steps of the considerations of the companies. Let us assume that the company *i* expects in the first step of the considerations that the unit price of the product $-i$ will be $p_{-i} = I_M$[12], where I_M are financial resources that demand spends on the market M. We will show later that the first expectation of the company i does not have any effect to the outcome, so to the equilibrium. But we remind here that the rational company i would never expect $p_{-i} < c_{-i}$. If $c_i < \frac{A_i}{A_{-i}} I_M$, rational company i will set $p_i = \frac{A_i}{A_{-i}} I_M - \Delta_1, \Delta_1 > 0 \wedge \Delta_1 < \frac{\frac{A_i}{A_{-i}} I_M \left(\frac{A_i}{A_{-i}} I_M - c_i\right)}{\frac{A_i}{A_{-i}} I_M + c_i}$, $\Delta_1 = \min_{\Delta > 0} \Delta$.

The second step of the consideration will be the turn of the company $-i$. Rational company $-i$ can and will expect the consideration of the company i from the first step. So

if $c_{-i} < I_M - \frac{A_{-i}}{A_i} \Delta_1, \Delta_1 > 0 \wedge \Delta_1 < \frac{\frac{A_i}{A_{-i}} I_M \left(\frac{A_i}{A_{-i}} I_M - c_i\right)}{\frac{A_i}{A_{-i}} I_M + c_i}, \Delta_1 = \min_{\Delta > 0} \Delta$,

company $-i$ will not set $p_{-i} = I_M$,

but $p_{-i} = I_M - \frac{A_{-i}}{A_i} \Delta_1 - \Delta_2, \Delta_2 > 0 \wedge \Delta_2 < \frac{\left(I_M - \frac{A_{-i}}{A_i} \Delta_1\right)\left(I_M - \frac{A_{-1}}{A_i} \Delta_1 - c_{-i}\right)}{I_M - \frac{A_{-i}}{A_i} \Delta_1 + c_{-i}}$, $\Delta_2 = \min_{\Delta > 0} \Delta$

under the expectation

$$p_i = \frac{A_i}{A_{-i}} I_M - \Delta_1, \Delta_1 > 0 \wedge \Delta_1 < \frac{\frac{A_i}{A_{-i}} I_M \left(\frac{A_i}{A_{-i}} I_M - c_i\right)}{\frac{A_i}{A_{-i}} I_M + c_i}, \Delta_1 = \min_{\Delta > 0} \Delta.$$

The third step of the considerations will be the turn of the company i again and analogically will lead to decision to set the unit price slightly lower than the relative unit price of the competitor's product by introducing another $\Delta_3 = \min_{\Delta > 0} \Delta$, until the unit variable costs c_i are lower than the unit price p_i.

We can observe that every consideration step makes the unit price lower until it reaches the level of the unit variable costs. Therefore, our assumption would be that after all consideration steps of the companies the company with relatively higher or equal unit variable costs will set the unit price of its product to the level of its unit variable costs.

If we can prove this assumption, we will get the same equilibriums that we have obtained from the Nash equilibrium approach.

[12] We assume that the first expectation of the considerations is that the competitor is willing to set the unit price on the "very high" level.

To prove it, let us slightly change the denotation of the companies without loss of generality. Let us denote the company $-i \in I$, $-i \neq i$ to be the company that makes decision in the first step of the consideration. Let us assume that the company $i \in I$ is the company that has relatively higher or equal unit variable costs on the market M.

Let us denote the final price of the company i $p_i^* = I_M - \Delta_i^*$, where I_M are financial resources that the demand spends on the market M. We use I_M here as the starting expectation of the company $-i$ about the price of the product i in the first step of the considerations of the companies.[13]

Let

$$\Delta_i^* = \sum_{j=1}^{\infty} \left(\frac{A_i}{A_{-i}}\right)^{\frac{(-1)^{j-1}+1}{2}} \Delta_j,\ j \in \mathbb{N},\ \Delta_j > 0 \wedge \Delta_j <$$

$$\frac{\left(\left(\frac{A_{-i}}{A_i}\right)^{\frac{(-1)^{j-1}+1}{2}} I_M - \sum_{k=1}^{j-1}\left(\frac{A_i}{A_{-i}}\right)^{\frac{(-1)^j-(-1)^k}{2}} \Delta_k\right)\left(\left(\frac{A_{-i}}{A_i}\right)^{\frac{(-1)^{j-1}+1}{2}} I_M - \sum_{k=1}^{j-1}\left(\frac{A_i}{A_{-i}}\right)^{\frac{(-1)^j-(-1)^k}{2}} \Delta_k - \left(\frac{(-1)^{j-1}+1}{2}\right) c_{-i} - \left(\frac{(-1)^j+1}{2}\right) c_i\right)}{\left(\frac{A_{-i}}{A_i}\right)^{\frac{(-1)^{j-1}+1}{2}} I_M - \sum_{k=1}^{j-1}\left(\frac{A_i}{A_{-i}}\right)^{\frac{(-1)^j-(-1)^k}{2}} \Delta_k + \left(\frac{(-1)^{j-1}+1}{2}\right) c_{-i} + \left(\frac{(-1)^j+1}{2}\right) c_i}\ \forall j \in \mathbb{N}, \quad (5.2.1)$$

Δ_i^* is deductively constructed from the consideration steps of the companies and the statement "after all considerations of the companies" is transformed to infinite number of the considerations.

It is possible to prove

$$\Delta_i^* = \sum_{j=1}^{\infty} \left(\frac{A_i}{A_{-i}}\right)^{\frac{(-1)^{j-1}+1}{2}} \Delta_j = I_M - c_i, \quad (5.2.2)$$

under the conditions $\Delta_j > 0 \wedge \Delta_j <$

$$\frac{\left(\left(\frac{A_{-i}}{A_i}\right)^{\frac{(-1)^{j-1}+1}{2}} I_M - \sum_{k=1}^{j-1}\left(\frac{A_2}{A_1}\right)^{\frac{(-1)^j-(-1)^k}{2}} \Delta_k\right)\left(\left(\frac{A_1}{A_2}\right)^{\frac{(-1)^{j-1}+1}{2}} I_M - \sum_{k=1}^{j-1}\left(\frac{A_2}{A_1}\right)^{\frac{(-1)^j-(-1)^k}{2}} \Delta_k - \left(\frac{(-1)^{j-1}+1}{2}\right) c_{-i} - \left(\frac{(-1)^j+1}{2}\right) c_i\right)}{\left(\frac{A_1}{A_2}\right)^{\frac{(-1)^{j-1}+1}{2}} I_M - \sum_{k=1}^{j-1}\left(\frac{A_2}{A_1}\right)^{\frac{(-1)^j-(-1)^k}{2}} \Delta_k + \left(\frac{(-1)^{j-1}+1}{2}\right) c_{-i} + \left(\frac{(-1)^j+1}{2}\right) c_i}\ \forall j \in$$

$\mathbb{N}$ and $c_i \geq \frac{A_i}{A_{-i}} c_{-i}$.

Proving this we prove $p_i^* = c_i$ under the condition $c_i \geq \frac{A_i}{A_{-i}} c_{-i}$.

The proof distinguishes three situations $A_i > A_{-i}$, $A_i = A_{-i}$ and $A_i < A_{-i}$. For every situation, it is possible to prove that the series $\sum_{j=1}^{\infty} \left(\frac{A_i}{A_{-i}}\right)^{\frac{(-1)^{j-1}+1}{2}} \Delta_j$ is convergent by the direct comparison test for convergence, because it is possible

[13] We will show later that the starting expectation is not important for the final outcomes.

to find mayorant convergent series to the series in (5.2.2). This mayorant convergent series to the series in (5.2.2) is derived from the positivity and upper boundary of Δ_j in (5.2.2).

Because $\sum_{j=1}^{\infty}\left(\frac{A_i}{A_{-i}}\right)^{\frac{(-1)^{j-1}+1}{2}}\Delta_j$ is convergent, then

$$\lim_{j\to\infty}\left(\frac{A_i}{A_{-i}}\right)^{\frac{(-1)^{j-1}+1}{2}}\Delta_j = 0.\text{[14]} \qquad (5.2.3)$$

Using (5.2.3) it is possible to find for every situation $A_i > A_{-i}$, $A_i = A_{-i}$ and $A_i < A_{-i}$ convergent series that defines $\sum_{j=1}^{\infty}\left(\frac{A_i}{A_{-i}}\right)^{\frac{(-1)^{j-1}+1}{2}}\Delta_j = I_M - c_i$ from the necessary condition for series convergence.

Finally, we can use lemma from the monotone convergence theorem of the sequence: if a sequence of real numbers is increasing and bounded above, then its supremum is the limit (Bibby, 1974).

The sequence of the partial sums of the series in (5.2.2) is increasing, because $\left(\frac{A_i}{A_{-i}}\right)^{\frac{(-1)^{j-1}+1}{2}}\Delta_j > 0\ \forall j \in \mathbb{N}$. We know that this sequence is bounded above, because $p_i^* = I_M - \Delta_i^* \geq c_i$, so the $\Delta_i^* = \sum_{j=1}^{\infty}\left(\frac{A_i}{A_{-i}}\right)^{\frac{(-1)^{j-1}+1}{2}}\Delta_j \leq I_M - c_i$, but because $\left(\frac{A_i}{A_{-i}}\right)^{\frac{(-1)^{j-1}+1}{2}}\Delta_j > 0\ \forall j \in \mathbb{N}$, then $\sum_{j=1}^{N}\left(\frac{A_i}{A_{-i}}\right)^{\frac{(-1)^{j-1}+1}{2}}\Delta_j \leq I_M - c_i\ \forall N \in \mathbb{N}$.

As we have said before, it is possible to prove that the series in (5.2.2) is convergent, so there exists the finite limit of the partial sums that equals the sum of the series in (5.2.2). Therefore, the limit of the sequence has to be equal to the sum of the series in (5.2.2). It is possible to show that the sequence of the partial sums of the series in (5.2.2) can reach its upper boundary, the $I_M - c_i$. If the sequence can reach its upper boundary, then this boundary is the maximum of the increasing positive sequence. Then, if increasing sequence can reach the maximum, then this maximum is the supremum. So the supremum of the sequence of the partial sums of the series in (5.2.2) is the $I_M - c_i$, so the limit of this sequence is $I_M - c_i$. Finally, the sequence of the partial sums of the series

[14] From the necessary condition for series convergence.

(5.2.2) can reach its limit and the limit of the sequence equals the sum of the series in (5.2.2), so the sum of the series $\sum_{j=1}^{\infty}\left(\frac{A_i}{A_{-i}}\right)^{\frac{(-1)^{J-1}+1}{2}}\Delta_j = I_M - c_i$.

So the final price of the product of the company *i* with "relatively higher" or "relatively equal" unit variable costs, after all considerations of the companies on the market M, will be the variable costs of the company *i*, that means $p_i = c_i$.

We will obtain the equilibrium from the best response function of the company –*i*. We know the final price of the company *i*, so we only need to use this final price in the best response function of the company –*i* that could be one of the (4.3.1), or (4.3.2).

If $c_i = \frac{A_i}{A_{-i}}c_{-i}$, then $c_{-i} = \frac{A_{-i}}{A_i}c_i$, so the company –*i* will set $p_{-i} = c_{-i} = \frac{A_{-i}}{A_i}c_i$ and we get equilibrium $p_i = c_i$, $p_{-i} = c_{-i}$ with demands $x_i = \frac{I_M}{2c_i}$, $x_{-i} = \frac{I_M}{2c_{-i}}$. If $c_i > \frac{A_i}{A_{-i}}c_{-i}$, then $c_{-i} < \frac{A_{-i}}{A_i}c_i$, so the company –*i* will set $p_{-i} = \frac{A_{-i}}{A_i}c_i - \Delta^*_{-i}, \Delta^*_{-i} > 0 \wedge \Delta^*_{-i} < \frac{\frac{A_{-i}}{A_i}c_i\left(\frac{A_{-i}}{A_i}c_i - c_{-i}\right)}{\frac{A_{-i}}{A_i}c_i + c_{-i}}, \Delta^*_{-i} = \min_{\Delta>0}\Delta$ and we get the equilibrium $p_i = c_i$, $p_{-i} = \frac{A_{-i}}{A_i}c_i - \Delta^*_{-i}, \Delta^*_{-i} > 0 \wedge \Delta^*_{-i} < \frac{\frac{A_{-i}}{A_i}c_i\left(\frac{A_{-i}}{A_i}c_i - c_{-i}\right)}{\frac{A_{-i}}{A_i}c_i + c_{-i}}, \Delta^*_{-i} = \min_{\Delta>0}\Delta$ with demands $x_i = 0$, $x_{-i} = \frac{I_M}{\frac{A_{-i}}{A_i}c_i - \Delta^*_{-i}}, \Delta^*_{-i} > 0 \wedge \Delta^*_{-i} < \frac{\frac{A_{-i}}{A_i}c_i\left(\frac{A_{-i}}{A_i}c_i - c_{-i}\right)}{\frac{A_{-i}}{A_i}c_i + c_{-i}}, \Delta^*_{-i} = \min_{\Delta>0}\Delta$.

Because we have not specified the company i with "relatively higher" or "relatively equal" unit variable costs, it could be any of the company $i \in I$. And as you can see the equilibriums are defined independently on the starting expectation in the step 1, so independently on the I_M. Therefore, it is not important what is the starting expectation in the first step of the considerations of the companies. This implies as well that it is not important, which company has the starting step.

5.3 Equilibrium of the market M with LUF

We have used two concepts to derive the equilibrium of the market M, the rational profile of the unit prices of the products on the market M. We have used the Nash equilibrium approach and the modeling of the considerations of the companies' approach. Both concepts have led us to the same equilibrium based on the conditions of the relative relationship of the unit variable costs of the companies. We present these equilibriums in the table below, distinguishing company $i \in I$ and company $-i \in I$, $-i \neq i$ as the competitor of the company i on the market M.

Table 1: Equilibrium of the Duopoly with Linear Utility Function of the Demand

Relative unit variable costs relationship	$c_i > \frac{A_i}{A_{-i}} c_{-i}$	$c_i = \frac{A_i}{A_{-i}} c_{-i}$	$c_i < \frac{A_i}{A_{-i}} c_{-i}$
Equilibrium unit price of the product *i* (p_i)	c_i	c_i	$\frac{A_i}{A_{-i}} c_{-i} - \Delta_i^*$ ***
Equilibrium unit price of the product –*i* (p_{-i})	$\frac{A_{-i}}{A_i} c_i - \Delta_{-i}^*$ ***	c_{-i}	c_{-i}
Number of demanded units of the product *i* (x_i)	0	$\frac{I_M}{2c_i}$	$\frac{I_M}{\frac{A_i}{A_{-i}} c_{-i} - \Delta_i^*}$ ***
Number of demanded units of the product –*i* (x_{-i})	$\frac{I_M}{\frac{A_{-i}}{A_i} c_i - \Delta_{-i}^*}$ ***	$\frac{I_M}{2c_{-i}}$	0

Profit of the company *i* (π_i)	$-fi$	$-fi$	$I_M \frac{\frac{A_i}{A_{-i}} c_{-i} - \Delta_i^* - c_i}{\frac{A_i}{A_{-i}} c_{-i} - \Delta_i^*} - f_i$ ***
Profit of the company – *i* (π_{-i})	$I_M \frac{\frac{A_{-i}}{A_i} c_i - \Delta_{-i}^* - c_{-i}}{\frac{A_{-i}}{A_i} c_i - \Delta_{-i}^*} - f_i$ ***	$-f\text{-}i$	$-f\text{-}i$
*** Δ* definition	$\Delta_{-i}^* > 0 \wedge \Delta_{-i}^* < \frac{\frac{A_{-i}}{A_i} c_i \left(\frac{A_{-i}}{A_i} c_i - c_{-i}\right)}{\frac{A_{-i}}{A_i} c_i + c_{-i}}$, $\Delta_{-i}^* = \min_{\Delta>0} \Delta$	Not applicable	$\Delta_i^* > 0 \wedge \Delta_i^* < \frac{\frac{A_i}{A_{-i}} c_{-i} \left(\frac{A_i}{A_{-i}} c_{-i} - c_i\right)}{\frac{A_i}{A_{-i}} c_{-i} + c_i}$, $\Delta_i^* = \min_{\Delta>0} \Delta$

Source: Author's own calculations.

We assume the fixed costs of the companies on the market M to be strictly positive, $f_i > 0 \ \forall \ i \in I$. Therefore, we can see in the Table 1 that the company, under the assumption of the rationality of the companies, can reach positive profit on the market M only if it has relatively lower unit variable costs. But in this case, the company with relatively higher unit variable costs achieves negative profit. In the case, when both companies have relatively equal unit variable costs, both companies reach negative profit.

6 Conclusion

Linear utility function of the demand represents assumption that the products on the market are perfect substitutes. But we can see that at least one company on the market with linear utility function of the demand achieves negative profit under the assumption of rationality of the companies. The assumption of rationality of the companies is a natural assumption, we can really expect real companies to be rational in their decisions. There is another natural assumption as well that companies on real markets are willing to maximize their profit.[15]

[15] We consider standard companies that have as their main goal the maximization of the profit. We are not talking about special companies, e.g. state owned companies that produce some specific product that meets some specific need of the population that is naturally not profitable.

We can expect that real company that is able to reach only negative profit will leave the market in the long term. But if products on real market were perfect substitutes, then these markets would not last in the long term. In the long term, these markets would disappear by the withdrawal of the company with only negative profit potential. But this is something what is not on a standard basis seen on real markets. There are many real markets that have strongly substitutive products and that last for a long time.

It is possible to analyze repeated situation of the market M using the Nash equilibrium concept, which would represent the long term interaction of the companies on the market M. It is possible to identify one outcome analyzing the repeated situation of the market M that enables both companies to reach positive profit. But this outcome is based on some more specific assumptions. The first assumption is that the companies expect the market M to last to infinity, or at least as long as the expected future profits from selecting higher price are greater than the actual profit. Secondly, companies have to select the strategies that are practically cartel strategies with potential ability to punish their competitor for deviance from the cartel agreement. But it is not rational to expect that the majority of real longterm markets of strongly substitutive products are cartels. There exist many barriers for companies to create cartels on real markets to protect the consumer.

Therefore, our conclusion should be that the model of the duopoly with linear utility function of the demand, so the assumption of perfectly substitutive products is applicable only for the short term real markets. The outcomes from the model are significantly different to the outcomes of the real long lasting markets with highly substitutive products.

We will have to choose a different model to analyze long lasting real markets with highly substitutive products. Potentially, a change in the utility function, which means the change in the assumption of perfectly substitutive products to imperfectly substitutive products, could bring interesting outcomes, more similar to the real ones. The candidate of the utility function of the demand to be used to model real long lasting markets could be quasi-linear utility function with simple element of the complementarity of the products.

6.1 Economic aspects of the duopoly with LUF

As we have seen, the model of the duopoly with linear utility function of the demand has limited application on the real markets. Assumption that the products are perfect substitutes lead to the outcome that can be seen only rarely, if ever, on long lasting markets of substitutive products.

On the other hand, this model could be applied to analyze short term real markets, where quality and price are the main drivers of the unrepeated decision of the demand. These short term unrepeated real markets can be represented by public or private procurements of some specific product or service, where companies compete to be selected to provide this product or service that is specifically defined just for the exact procurement. We can look at these real procurements as unique competitions of the companies, i.e. unique unrepeated markets.

Assumption that the products are perfect substitutes on some market has its origin in economic theory. The perfect competition, the idealistic market model, inter alia assumes the homogeneity of the products (Varian, 1995). Homogeneity of the products is assumption that the products on the markets are the same from the demand (consumer) point of view. They are the same in ability to meet some specific need, quality, availability, knowingness, marketing, and in any other aspects of the product. Homogeneity is more specific assumption than the assumption of perfect substitution of the products. It is only one case of the perfect substitution that can be represented by the linear utility function without the coefficients of the attractiveness, or with the same coefficients of the attractiveness ($A_1 = A_2$ in the case of duopoly).

We can find assumption of the homogeneity in early market models as well. The Cournot model and the Bertrand model assume the homogeneity of the products (Varian, 1995; Cournot, 1838; Bertrand, 1883; Edgeworth, 1889). The Bertrand model was created as the criticism of the Cournot model, so it just took over this assumption. Cournot had created his model on the example of the market of the spring water. The market of the spring water represents market with highly homogeneous products, especially in times of Cournot.

We believe that by softening the assumption of the homogeneity of the products to the perfect substitution, we are getting closer to the real markets, and we are making application of the model to be more general. But as we have seen, the model with linear utility function of the demand (the assumption of the perfect substitution) has still limited application. Softening the assumption of the perfect substitution of the products to the imperfect substitution could define model that could be used to analyze more general real markets.

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