SPATIAL COMPETITION WITH CONSIDERING THE WEIGHT OF THE NODES: CASE STUDY

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Priestorová konkurencia so zohľadnením váhy uzlov: prípadová štúdia

Abstract: *The matrix games are a part of the cornerstones of the game theory, generally used to explain the competition models where the competitors are two or more entities (players). The paper is focused on using the models of game theory to find the optimal location of the player while taking into consideration the customer behavior. In the paper we model a situation of two companies competing for a market which is divided into different nodes. Both companies offer identical goods on the market, but the price of these goods may vary. The players decide on moving to a selected location in order of gain more clients, considering the cost of consumers, which includes both the price of the goods and the transport costs. As a result, the companies are focused on the place close to more possible clients. A spatial competition model is solved using matrix games in a case study of Bratislava city and the results are compared with a model when the weight of nodes is not considered.*

Keywords: *Spatial competition, matrix game, weighted node.*

JEL Classification: L16, R41

1. Introduction

The first known location models are associated with the name of Hotelling [4]. Hotelling considered the location along the line and in our analysis we follow these considerations by adding a spatial structure that is described as a graph as in Sequeira Lopez & Čičková [8]. In order to extend such analysis, we consider a specific city where every district has a different number of residents. As a result, the companies decide their location in the nodes with a higher potential demand. Of course, the decision of the companies in this case will be influenced by the decisions of the consumers. In other studies of spatial games the consumers are divided into two groups, myopic (short-sighted), that is, they are unaware of the damage that occurs when purchasing goods at a location other than the location preferred by some regulator.

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Such behaviour can cause negative externalities because of, for example, higher traffic loads that cause higher pollution of the entire area and not just at a given location [8]. If the regulator's preference for a given node is to support local manufacturers or suppliers, ignoring preferred nodes can have a negative impact on the economic situation of the entire region. If a regulator wants to support the growth of some area where there is not a high level of conglomeration, a zoning process can be used as shown in Sequeira Lopez [7]. Generally, zoning is used as a way to isolate negative externalities (such as pollution) outside public areas or in the case of product (service) promotion in the preferred area [6].

In our model below, the participation of the regulator will not be taken into consideration. In our analysis we compare the situation in which the node is taken as unity and when the nodes are taken as a space with different population.

We will solve the situation mentioned above by the game theory models. The model could be formulated as a game with a constant sum if we consider that there are only two sellers in the market and consumers will take into account the cost of transport from one node to other in the moment of purchasing of a purchase. Therefore, we will introduce the basic formalization of such a game.

1.1 Constant-Sum Game

A two-player game has two participants and each player has several possible variants to maximize their payoffs, that is, each player chooses independently (without information about the opponent's choice) one of the final number of scenarios (strategies). The model is based on the assumption that players are intelligent and want to achieve the best result.

In constant sum games (matrix games), it is assumed that the interests of the players are diametrically opposed, antagonistic, which means that one player winning means the other player losing.[2]

On the contrary, games with a non-constant sum (bimatrix games), so-called non-antagonistic games, analyse such conflict situations where one player winning does not automatically mean the other player losing [3]. The goal is to find out what strategy a player has to choose so that choosing a different strategy can't improve his outcome [5]. Such kind of games could be solved by finding the strategies which meet the Karush-Kuhn-Tucker conditions [1].

[2] In our paper we do not deal with cases with conflicting situations of *n* players, although these situations are already elaborated in the literature.

A two-player game is based on the following formula: Let $P = \{1,2\}$ be a set of players where each player has a finite set of strategies (X – player 1, Y – player 2}, that is, player 1 selects $x \in X$, player 2 selects $y \in Y$. The set of all game results can be defined as $(x,y) \in X, Y$. Elements of set X and Y can be ordered by a finite number of nonnegative numbers (elements of set $X: i = 1,2, \dots m$ and elements of set $Y: j = 1,2, \dots n$). The values of the game for player 1 are included in the matrix $\mathbf{A}_{mxn} = \{a_{ij}\}$, where a_{ij}denotesthe player's payoff on the result (i, j), that is, if $a_{ij} > 0$, the player gains and if$a_{ij} < 0$, the player pays. On the other hand, the values of the game for player 2 are included in the matrix$\mathbf{B}_{nxm} = \{b_{ji}\}$, where$b_{ji}$ indicates the player's payoff on result (j, i).

Constant sum can be characterized as follows:

$$\mathbf{B}_{nxm} = C_{nxm} - \mathbf{A}^{T}_{mxn}, \text{ where} C_{nxm} = \{c\}, \qquad (1)$$

where$c$ is a constant independent on the selection strategy.

The solution of the game is a formulation of equilibrium strategies for both players. A player's state is defined as equilibrium if the system tends to remain in that state under certain conditions. Only such a set of strategies can be considered a satisfactory outcome if any unilateral infringement effort automatically leads to a loss for the player trying to do so.

The solutions to the above mentioned game are based on the following assumptions: Both players have complete information about the conflict model, that is, they know the payoff matrix$A_{mxn} = \{a_{ij}\}$, players are intelligent, so they want to maximize their payoff and know that the opponent will do so, and players are careful and try to minimize their risk. The solution to the game is to identify the equilibrium point in pure strategies [3, 2].

2. Spatial Competition and a Graph with Weighted Nodes

A spatial game is based on the following assumptions: Let $V = \{1,2, \dots n\}$ be a set of consumers and let be given a finite continuous oriented edgewise-rated graph $G = (V, H)$, where V represents a non-empty n-element set of graph nodes, and $H \subset VxV$ represents a set of edges $h_{ij}, i,j \in V$ from the *i-th* to *j-th*, with each oriented edge h_{ij} being assigned a real number $o(h_{ij})$ called a valuation or also the edge value of h_{ij}. The network game is formulated in a so-called complete or a complete weighted graph $\bar{G} = (V, \bar{H})$ with the same set of nodes as graph G, where $\bar{H}$ is the set of edges between each pair of nodes i and j, their valuation being equal to the minimum distance between the nodes i and j in the original graph $i,j \in V$. If d_{ij} represents the minimum distance (the shortest path length) between nodes i and j, then the matrix $\mathbf{D}_{nxn} = \{d_{ij}\}$ is the shortest distance matrix.

Let´s assume two companies (players), $P = \{1,2\}$ offering a homogeneous product (goods or service) that have the ability to build their operations in one of the nodes of graph $\bar{G}$. Suppose the nodes of graph also represent the seat of the consumers with constant demand. Although both players offer an identical product in an unlimited amount, the product price is different. Let us denote $p^{(1)}$ the product price for player 1 and $p^{(2)}$ the product price for player 2. Consumers take into consideration the total price of the product consisting of both the purchase price of the product and the price of the transport to a chosen company. Transport costs are rated as t/unit of distance. The aim is to identify those nodes in which companies build their operations, assuming mutual interaction, and it is known that customers always prefer lower cost purchases (in case of equal costs, companies will split demand in half). The model taking into account the above assumptions was presented in Sequeira Lopez & Čičková [8]. In this way, the cost of the consumer of purchasing at company 1 can be written in a matrix $\mathbf{N}^{(1)} = \{n_{ij}^{(1)}\}, i,j \in V$ with elements are defined as follows:

$$n_{ij}^{(1)} = t * d_{ij} + p^{(1)}, i,j \in V \tag{2}$$

Analogically for the player 2 we specify the matrix $\mathbf{N}^{(2)} = \left\{n_{ij}^{(2)}\right\}, i,j \in V$:

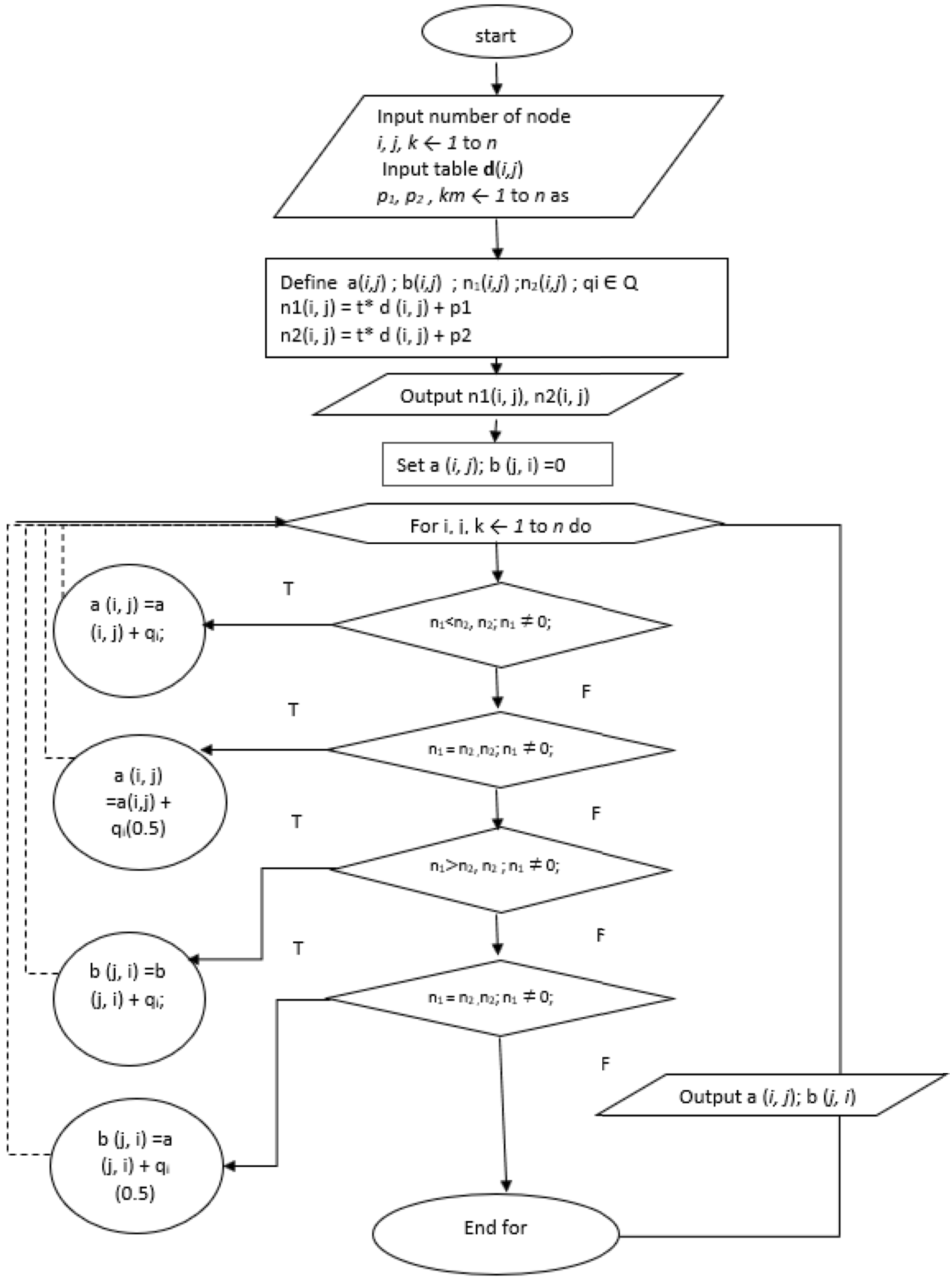
$$n_{ij}^{(2)} = t * d_{ij} + p^{(2)}, i,j \in V \tag{3}$$

Let’s introduce a parameter $q_i \in Q,\ i = \{1,2,\dots n\}$, that represents the weight of elements in a given node, i.e. in this case it will represent the number of residents in a given node. As a result, we will not count a node as in Sequeira Lopez & Čičková [8], but in this case the companies will consider the number of residents relating to potential demand as a determinant factor for investment in a given node. By introducing such factor we approach more to the real competition scenarios.

Factors (2) and (3) take into account both the total transport costs of the transport from the $i-th$ customer to the company as well as the purchase price of the product.

Furthermore, we assume the following: If player 1 builds a service place in the $j-th$ node, he will get the customer from the $i-th$ node only if $n_{ij}^{(1)} < n_{ij}^{(2)}$, $i,j \in \{1,2,\dots n\}$; if $n_{ij}^{(1)} = n_{ij}^{(2)}$ the players share the demand equally, otherwise the customer from the $i-th$ node is served by player 2. We do not consider lost demand in this case, i.e., a consumer does his purchase in a place closer to him. Then it is possible to define the elements of the payoff matrix of player 1 (**A**) and player 2 (**B**) in the form of the following flowchart:

Chart 1



Source: Own presentation

In the graph we put also the value of b_{ji} which is the payoff matrix of player 2 but in the rest of the paper we will work only with matrix a_{ij}, the matrix b_{ji} will follow the equation (1) described above.

In the next section, we describe this approach by solving illustrative examples and the result of both models will be compared (the model with non-unity value of node and the model of q_i value of node).

2.1 Case Study of the City of Bratislava

Bratislava is the capital of the Slovak Republic, by its size and population it is the largest city in Slovakia; it officially has approximately 430 000 inhabitants. It is a border town, as some of its districts lie on the border with Austria and Hungary.

Also in Bratislava, there has been an increase in the number of shopping centres in various city districts in the last twenty-five years, which are in our view and in terms of location more focused on domestic customers. For the purposes of our analysis, we focused primarily on whether it is possible to apply zoning in Bratislava.

An important parameter for the purposes of our analysis is the number of residents of urban areas, i.e. a number of people residing in the city.

The city of Bratislava is comprised by seventeen city districts. The following table shows a list of the districts according to the number of residents.

Table 1

Number	City district	District	Area in m²	Population
1.	Petržalka	Bratislava V	28 680 130	111 778
2.	Ružinov	Bratislava II	39 700 472	72 360
3.	Staré Mesto	Bratislava I	9 590 189	41 086
4.	Nové Mesto	Bratislava III	37 481 490	38 038
5.	KarlovaVes	Bratislava IV	10 947 890	34 772
6.	Dúbravka	Bratislava IV	8 648 849	34 745
7.	Rača	Bratislava III	23 659 322	20 660
8.	Vrakuňa	Bratislava II	10 296 687	19 987
9.	Podunajské Biskupice	Bratislava II	42 492 965	21 417
10.	Devínska Nová Ves	Bratislava IV	24 224 539	16 227
11.	Lamač	Bratislava IV	6 542 382	6 804
12.	Vajnory	Bratislava III	13 500 000	5 168
13.	Záhorská Bystrica	Bratislava IV	32 297 824	3 422
14.	Rusovce	Bratislava V	25 558 272	2 751
15.	Jarovce	Bratislava V	21 342 463	1 455
16.	Devín	Bratislava IV	13 964 215	1 122
17.	Čunovo	Bratislava V	18 622 751	1 009

Data of December 31, 2017,
Source: http://www.sodbtn.sk/obce/okres_ob.php?kod_okresu=103

It is estimated that several thousand more people arrive in the city than stated in the statistical outputs, mainly working people and students. But even if these data are biased, for the purpose of our analysis it will be sufficient to assume that the companies focus more on stable clients and the commuting people are not considered a decisive factor.

The data of population in each city district can be very important for the final decision of the company; we will include it in models where we consider different levels of demand. Demand will be represented by the number of residents in the given nodes (city districts).

Based on the location of the city districts of Bratislava we can calculate the shortest distance matrix. The result will be a matrix and also a graph that will serve as the basic elements of our analysis.

The shortest distance matrix of Bratislava city districts

$\mathbf{D} = d_{ij}, i, j = 1,2, \ldots 17$ is indicated in the following table:

Table 2

Uzly	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	6	10	7	9	10	14	13	21	21	18	17	20	16	17	10	16
2	6	0	6	13	6	4	8	10	15	18	15	11	16	12	13	16	22
3	10	6	0	17	12	2	14	16	21	24	21	9	10	6	7	20	26
4	7	13	17	0	16	17	21	20	28	28	25	24	27	23	24	3	9
5	9	6	12	16	0	10	8	4	15	12	9	17	22	18	19	19	25
6	10	4	2	17	10	0	12	14	19	22	19	7	12	8	9	20	26
7	14	8	14	21	8	12	0	4	7	12	16	19	24	20	21	24	30
8	13	10	16	20	4	14	4	0	11	8	12	21	26	22	23	23	29
9	21	15	21	28	15	19	7	11	0	7	11	26	31	27	28	31	37
10	21	18	24	28	12	22	12	8	7	0	4	29	34	30	31	31	37
11	18	15	21	25	9	19	16	12	11	4	0	26	31	27	28	28	34
12	17	11	9	24	17	7	19	21	26	29	26	0	5	15	16	27	33
13	20	16	10	27	22	12	24	26	31	34	31	5	0	16	17	30	36
14	16	12	6	23	18	8	20	22	27	30	27	15	16	0	3	26	32
15	17	13	7	24	19	9	21	23	28	31	28	16	17	3	0	27	33
16	10	16	20	3	19	20	24	23	31	31	28	27	30	26	27	0	6
17	16	22	26	9	25	26	30	29	37	37	34	33	36	32	33	6	0

Source: Own presentation

The graph shows all 17 nodes of the city of Bratislava. The shortest distance matrix and the graph were calculated using Matlab software. Matlab allows you to calculate a matrix using a minimum number of lines of code thanks to the wide range of libraries that this program offers and thus with just a few lines of code we can calculate a matrix of shortest distances from start node to all nodes in the graph.

Consequently, based on (2), (3) we can quantify elements of the total costs matrix for player 1, **N**(1) and for player 2,**N**(2), assuming unit transport costs $t = 1$. We assume that each player offers a homogeneous product, the product price of player 1 (company A) is $p^{(1)}$= 8.8 and the product price of player 2 (company B) je $p^{(2)}$= 8.5.

The cost for **N**(1) are shown in the following table which include the cost of transportation and purchasing cost.

Table 3

Uzla	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	8.8	14.8	18.8	15.8	17.8	18.8	22.8	21.8	29.8	29.8	26.8	25.8	28.8	24.8	25.8	18.8	24.8
2	14.8	8.8	14.8	21.8	14.8	12.8	16.8	18.8	23.8	26.8	23.8	19.8	24.8	20.8	21.8	24.8	30.8
3	18.8	14.8	8.8	25.8	20.8	10.8	22.8	24.8	29.8	32.8	29.8	17.8	18.8	14.8	15.8	28.8	34.8
4	15.8	21.8	25.8	8.8	24.8	25.8	29.8	28.8	36.8	36.8	33.8	32.8	35.8	31.8	32.8	11.8	17.8
5	17.8	14.8	20.8	24.8	8.8	18.8	16.8	12.8	23.8	20.8	17.8	25.8	30.8	26.8	27.8	27.8	33.8
6	18.8	12.8	10.8	25.8	18.8	8.8	20.8	22.8	27.8	30.8	27.8	15.8	20.8	16.8	17.8	28.8	34.8
7	22.8	16.8	22.8	29.8	16.8	20.8	8.8	12.8	15.8	20.8	24.8	27.8	32.8	28.8	29.8	32.8	38.8
8	21.8	18.8	24.8	28.8	12.8	22.8	12.8	8.8	19.8	16.8	20.8	29.8	34.8	30.8	31.8	31.8	37.8
9	29.8	23.8	29.8	36.8	23.8	27.8	15.8	19.8	8.8	15.8	19.8	34.8	39.8	35.8	36.8	39.8	45.8
10	29.8	26.8	32.8	36.8	20.8	30.8	20.8	16.8	15.8	8.8	12.8	37.8	42.8	38.8	39.8	39.8	45.8
11	26.8	23.8	29.8	33.8	17.8	27.8	24.8	20.8	19.8	12.8	8.8	34.8	39.8	35.8	36.8	36.8	42.8
12	25.8	19.8	17.8	32.8	25.8	15.8	27.8	29.8	34.8	37.8	34.8	8.8	13.8	23.8	24.8	35.8	41.8
13	28.8	24.8	18.8	35.8	30.8	20.8	32.8	34.8	39.8	42.8	39.8	13.8	8.8	24.8	25.8	38.8	44.8
14	24.8	20.8	14.8	31.8	26.8	16.8	28.8	30.8	35.8	38.8	35.8	23.8	24.8	8.8	11.8	34.8	40.8
15	25.8	21.8	15.8	32.8	27.8	17.8	29.8	31.8	36.8	39.8	36.8	24.8	25.8	11.8	8.8	35.8	41.8
16	18.8	24.8	28.8	11.8	27.8	28.8	32.8	31.8	39.8	39.8	36.8	35.8	38.8	34.8	35.8	8.8	14.8
17	24.8	30.8	34.8	17.8	33.8	34.8	38.8	37.8	45.8	45.8	42.8	41.8	44.8	40.8	41.8	14.8	8.8

Source: Own presentation

And based on (3) the total cost **N**(2) are as follows:

Table 4

Uzla	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	8.5	14.5	18.5	15.5	17.5	18.5	22.5	21.5	29.5	29.5	26.5	25.5	28.5	24.5	25.5	18.5	24.5
2	14.5	8.5	14.5	21.5	14.5	12.5	16.5	18.5	23.5	26.5	23.5	19.5	24.5	20.5	21.5	24.5	30.5
3	18.5	14.5	8.5	25.5	20.5	10.5	22.5	24.5	29.5	32.5	29.5	17.5	18.5	14.5	15.5	28.5	34.5
4	15.5	21.5	25.5	8.5	24.5	25.5	29.5	28.5	36.5	36.5	33.5	32.5	35.5	31.5	32.5	11.5	17.5
5	17.5	14.5	20.5	24.5	8.5	18.5	16.5	12.5	23.5	20.5	17.5	25.5	30.5	26.5	27.5	27.5	33.5
6	18.5	12.5	10.5	25.5	18.5	8.5	20.5	22.5	27.5	30.5	27.5	15.5	20.5	16.5	17.5	28.5	34.5
7	22.5	16.5	22.5	29.5	16.5	20.5	8.5	12.5	15.5	20.5	24.5	27.5	32.5	28.5	29.5	32.5	38.5
8	21.5	18.5	24.5	28.5	12.5	22.5	12.5	8.5	19.5	16.5	20.5	29.5	34.5	30.5	31.5	31.5	37.5
9	29.5	23.5	29.5	36.5	23.5	27.5	15.5	19.5	8.5	15.5	19.5	34.5	39.5	35.5	36.5	39.5	45.5
10	29.5	26.5	32.5	36.5	20.5	30.5	20.5	16.5	15.5	8.5	12.5	37.5	42.5	38.5	39.5	39.5	45.5
11	26.5	23.5	29.5	33.5	17.5	27.5	24.5	20.5	19.5	12.5	8.5	34.5	39.5	35.5	36.5	36.5	42.5
12	25.5	19.5	17.5	32.5	25.5	15.5	27.5	29.5	34.5	37.5	34.5	8.5	13.5	23.5	24.5	35.5	41.5
13	28.5	24.5	18.5	35.5	30.5	20.5	32.5	34.5	39.5	42.5	39.5	13.5	8.5	24.5	25.5	38.5	44.5
14	24.5	20.5	14.5	31.5	26.5	16.5	28.5	30.5	35.5	38.5	35.5	23.5	24.5	8.5	11.5	34.5	40.5
15	25.5	21.5	15.5	32.5	27.5	17.5	29.5	31.5	36.5	39.5	36.5	24.5	25.5	11.5	8.5	35.5	41.5
16	18.5	24.5	28.5	11.5	27.5	28.5	32.5	31.5	39.5	39.5	36.5	35.5	38.5	34.5	35.5	8.5	14.5
17	24.5	30.5	34.5	17.5	33.5	34.5	38.5	37.5	45.5	45.5	42.5	41.5	44.5	40.5	41.5	14.5	8.5

Source: Own presentation

As a result, we can calculate the payoff matrix $a_{ij}, i, j = \{1,2 \dots 17\}$ of this game:

Table 5

Node	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1		117	204	428	236	204	336	336	370	370	343	255	293	255	255	428	429
2	316		255	428	336	255	372	336	405	377	412	407	407	314	391	428	428
3	178	178		316	178	119	336	336	370	336	343	407	407	391	391	316	428
4	5	5	117		117	117	117	117	336	336	336	204	255	204	204	429	432
5	97	87	214	316		214	372	372	406	412	412	255	293	255	255	428	428
6	229	178	197	316	219		336	336	370	370	343	407	407	391	391	316	428
7	97	61	97	316	10	97		229	412	412	415	255	255	255	255	316	428
8	97	97	97	316	61	97	204		412	412	412	255	255	255	255	316	428
9	62	28	62	97	26	62	21	21		229	45	97	138	97	97	197	316
10	62	56	62	97	21	56	17	21	87		61	97	97	97	97	97	316
11	56	21	90	97	21	90	17	17	169	372		97	255	97	97	197	316
12	178	26	26	229	178	26	178	178	336	336	336		428	202	202	316	316
13	26	26	26	178	140	26	140	178	295	336	178	5		26	26	178	316
14	178	41	41	229	178	41	178	178	336	336	336	231	407		411	316	316
15	178	41	41	178	178	41	178	178	336	336	336	114	407	21		229	316
16	5	5	5	4	5	5	117	117	236	336	236	117	204	117	117		432
17	4	5	5	1	5	5	5	5	117	117	117	117	117	5	117	1	

Source: Own presentation

The results were calculated using Solver Lindo, version 24.9.2 r64480.

The following table shows optimal strategy for player 1 and a comparison of both models – a model with unit weight nodes and a model with different weight nodes.

Table 6

Model/strategic node	1	2	3	5	6	8
Basic model	0.161	0.027	0.210	0.007	0.094	0.501
Differentdemand	0.140	0.009	0.332	0.320	0.200	0

Source: Own presentation

Node 1 (city district Bratislava – Petržalka) actually has the largest population of all city districts, but when the model was extended by different demand, the strategic interest of this node decreased. This change could be explained by the fact that even though this node has the largest population, the nodes close to it have significantly lower population. Because of its geographical location, node 1 is far away from other nodes with significant population. Due to the real geographical location, this node could attract the demand of residents of node 4 (Bratislava – Jarovce), 16 (Bratislava – Rusovce), and 17 (Bratislava – Čunovo), and these nodes have the lowest population.

For node 3 (Ružinov), this effect is reversed; interest in this node increases as we add the weight of demand in each node.

At node 5 (Karlova Ves) there are also changes in preferences according to individual assumptions. In the basic model, this node will not be of interest to player 1. Conversely, after adding the weights of potential demand, interest in this node is increasing. Taking into account also the abundance of nearby nodes, Karlova Ves is an interesting node for player 1 for the location of its operation, for node 6 (Nové Mesto) the result is similar when adding the weight of demand.

Other nodes, e.g. node 8 (Dúbravka) resulted in zero after extending the basic model by adding different demand in city districts, which can be also explained by its location outside of the city.

The following table shows optimal strategy for player 2 and a comparison of both models – a model with unit weight nodes and a model with different weight nodes.

Table 7

Model/strategicnode	1	2	3	5	6	8
Basic model	0.071	0.513	0.051	0.126	0.199	0.041
Different demand	0.140	0.485	0.131	0.002	0.268	0

Source: Own presentation

Player 2 will respond to player 1's strategy so that if player 1 does not locate his centre in node 2 (Bratislava – Staré Mesto), then player 2 would see the opportunity to invest in that node in any case.

For node 1 (Petržalka), player 2 should invest every time according to the investment index similar to player 1. His investment would be beneficial in node 6 (Nové Mesto), where he would also gain potential demand from neighbouring nodes.

In case player 1 is located in node 5 (Bratislava –Karlova Ves), player 2 should consider applying a different strategy, such as investing in node 6 (Bratislava – Nové Mesto).

In the above mentioned examples, we do not consider that a consumer has a maximum price willing to pay for goods based on distance and shipping costs. This is a possible extension of the model and it is subject to author`s further research.

3. Conclusion

Spatial games are a specific kind of games in which the player corresponds to the strategy of the competitor, such specific problems can be solved by game theory models. The problem is formulated for duopoly (on the supply side). The issues are analysed in the transport network with individual buyers located in the individual nodes of such network. However, in this case, the nodes have different weights depending on population defined by residents in certain city districts. The sellers decide on their position while trying to respect the behaviour of consumers who minimize both the costs associated with the transport price and the transport costs. Based on the analysed models and illustrative examples we can see that if we change one of the factors in the model it will affect to the result of the game. The competitors have to take into account such results. The basic model was extended by introduction of the factor of conglomeration of population in every node of a graph. In our opinion, the described situation can be used in such situations where the competitors need to take into account every possible client, i.e., a grocery store, a car dealer, shoe stores, bakeries, etc. It is obvious that every consumer reacts to the location of the seller, and could set a maximum difference of price that they are willing to pay for purchase of a product as it is described in Sequeira Lopez & Čičková [8].

A case study using as an example the city of Bratislava and the different population of city districts illustrate the use of the game theory model. The GAMS professional software, which ranks among the powerful optimization computing environments, was used to solve the games mentioned above.

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